

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Number

Four operations with fractions (non-calculator)

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

$$\left(\frac{5}{6} =\right) \frac{10}{12}$$

or

converts both fractions to a common denominator with at least one correct

$$\text{eg } \frac{60}{72} (+) \frac{42}{72}$$

M1

$$\frac{17}{12}$$

oe improper fraction eg $\frac{102}{72}$

A1

$$1\frac{5}{12}$$

oe mixed number eg $1\frac{30}{72}$

ft correct conversion of an improper fraction to a mixed number

B1ft

Additional Guidance

$$1\frac{30}{72}$$

M1A1B1

$$1\frac{5}{12}$$

seen in working with $\frac{17}{12}$ on answer line

M1A1B0

$$\frac{17}{12} (=) 1\frac{5}{12}$$

(final answer is the mixed number)

M1A1B1

$$1\frac{5}{12} (=) \frac{17}{12}$$

(final answer is the improper fraction)

M1A1B0

[3]

Q2.

0.75

B1

[1]

Q3.

$$\frac{11}{4} (\times) \frac{12}{7}$$

Converts both fractions to improper with at least one correct

M1

$$\frac{\text{their } 11 \times \text{their } 12}{\text{their } 4 \times \text{their } 7} \text{ or } \frac{132}{28}$$

$$\text{or } 4\frac{20}{28} \text{ or } \frac{33}{7}$$

oe fraction

M1dep

$$4\frac{5}{7}$$

A1

[3]

Q4.

- (a) $200 \div 5$ or 40
or 200×3 or 600

$$\text{or } \frac{3}{5} \times 200$$

$$\text{or } 0.6 \times 200$$

120

M1

A1

- (b) Attempt to divide 12.6 by 2 or 6.3

32.1

M1

SC1 for 19.2

A1

[4]

Q5.

$$12 \times \frac{1}{2}$$

B1

[1]

Q6.

$$\frac{6}{18}$$

or

Converts both fractions to an appropriate common denominator with at least one correct numerator

$$\text{eg } \frac{33}{54} \text{ and } \frac{18}{54}$$

M1

$$\frac{5}{18}$$

$$\text{oe fraction eg } \frac{15}{54}$$

$$\text{SC1 } \frac{17}{18} \text{ (oe fraction)}$$

A1

Additional Guidance

Ignore incorrect attempt to simplify after correct answer seen

$$\frac{22}{30} \text{ and } \frac{10}{30}$$

not an appropriate denominator

M0

[2]

Q7.

$$\frac{2}{5} \text{ or } 0.4$$

oe

B1

Additional Guidance

Ignore attempt to simplify or convert if correct fraction seen

[1]

Q8.

$$\left(\frac{5}{8} = \right) \frac{10}{16}$$

or

Converts both fractions to a common denominator with at least one numerator correct

M1

$$\frac{23}{16}$$

oe improper fraction

A1

$$1\frac{7}{16}$$

oe mixed number

ft correct conversion of their improper fraction to a mixed number

B1ft

Additional Guidance

Ignore incorrect simplification after B1 or B1ft awarded

[3]

Q9.

$$2\frac{1}{4}$$

oe mixed number

B1

Additional Guidance

$$\frac{9}{4} = 2\frac{1}{4} \text{ or } 2.25 = 2\frac{1}{4} \text{ on answer line}$$

B1

$$2\frac{1}{4} = \frac{9}{4} \text{ or } 2\frac{1}{4} = 2.25 \text{ on answer line}$$

B0

Otherwise, $2\frac{1}{4}$ and $\frac{9}{4}$ or $2\frac{1}{4}$ and 2.25 on answer line in either order (or in working with answer line blank and answer unclear)

B0

$$1\frac{5}{4}$$

B0

$$2\left(\frac{1}{4}\right) \text{ or } 2 + \frac{1}{4}$$

B0

[1]

Q10.

$$1\frac{1}{7}$$

B1

[1]

Q11.

(a) $\frac{6}{7}$

B1

(b) $\frac{3}{8} \times \frac{10}{7}$

$$\frac{30}{56} \text{ or } \frac{15}{28}$$

M1

A1

[3]

Q12.

(a) $\frac{5}{20} (+) \frac{14}{20}$

oe fractions with a common denominator
and at least one correct numerator

M1

$$\frac{19}{20}$$

oe fraction eg $\frac{38}{40}$ or $\frac{95}{100}$
SC1 0.95

A1

(b) $\frac{3 \times 7}{5 \times 2}$ or $\frac{21}{10}$

oe fraction eg $\frac{210}{100}$

M1

$$2\frac{1}{10}$$

oe mixed number eg $2\frac{10}{100}$
SC1 2.1

A1

[4]

Q13.

$$2\frac{1}{2}$$

B1

[1]

Q14.

Alternative method 1

$$\frac{17}{2} \text{ or } \frac{8}{3}$$

oe fractions

M1

their $\frac{17}{2} \times$ their $\frac{3}{8}$

*conversion of both mixed numbers to improper fractions and
multiplication of the conversion of $8\frac{1}{2}$ by the reciprocal of the
conversion of $2\frac{2}{3}$*

M1

$$\frac{51}{16}$$

oe fraction or decimal

A1

$$3\frac{3}{16}$$

*oe mixed number
ft correct conversion of their improper fraction to a mixed
number*

B1ft

Alternative method 2

$$\frac{17}{2} \text{ or } \frac{8}{3}$$

oe fractions

M1

$$\frac{51}{6} \div \frac{16}{6}$$

*conversion of both mixed numbers to improper fractions,
correct conversion to improper fractions with a common
denominator and division of the conversion of $8\frac{1}{2}$ by the
conversion of $2\frac{2}{3}$*

M1

$$\frac{51}{16}$$

oe fraction or decimal

A1

$$3\frac{3}{16}$$

*oe mixed number
ft correct conversion of their improper fraction to a mixed
number*

B1ft

Additional Guidance

Working with decimals

0, 3 or 4

Ignore incorrect attempt to simplify a mixed number

eg $3\frac{3}{16} = 3\frac{1}{8}$

M1M1A1B1

$3\frac{3}{16}$ seen, then $\frac{51}{16}$ on answer line

M1M1A1B0

$\frac{9}{2}$ and $\frac{8}{3}$, $\frac{27}{6} \div \frac{16}{6}$, $\frac{27}{16}$, $1\frac{11}{16}$

M1M1A0B1ft

$\frac{9}{2}$ and $\frac{8}{3}$, $\frac{27}{6} \div \frac{16}{6}$, $1\frac{11}{16}$

M1M1A0B1ft

$\frac{9}{2}$ and $\frac{4}{3}$, $\frac{27}{6} \div \frac{8}{6}$, $\frac{27}{8}$, $3\frac{3}{8}$

M0M1A0B1ft

[4]

Q15.

(a) $\frac{2}{10} (+) \frac{7}{10}$

oe fractions with a common denominator and at least one correct numerator

M1

$\frac{9}{10}$

oe fraction eg $\frac{18}{20}$
SC1 0.9

A1

(b) $\frac{3 \times 7}{5 \times 2}$ or $\frac{21}{10}$

oe fraction eg $\frac{210}{100}$

M1

$2\frac{1}{10}$

oe mixed number eg $2\frac{10}{100}$
SC1 2.1

A1

[4]

Q16.

Alternative method 1

$$(1\frac{1}{4} =) \frac{5}{4}$$

oe improper fraction

M1

$$\frac{4}{8} \text{ and } \frac{10}{8} \text{ or } \frac{2}{4} \text{ and } \frac{5}{4}$$

or
 $\frac{3.5}{4}$

*oe common denominator with at least one correct numerator
may be seen as start and end of a list*

M1dep

$$\frac{7}{8}$$

oe fraction

A1

Alternative method 2

$$(1\frac{1}{4} - \frac{1}{2} =) \frac{3}{4}$$

oe

M1

$$\frac{1}{2} + \text{their } (\frac{3}{4} \div 2)$$

or

$$1\frac{1}{4} - \text{their } (\frac{3}{4} \div 2)$$

oe

M1dep

$$\frac{7}{8}$$

oe fraction

A1

Alternative method 3

$$(1\frac{1}{4} + \frac{1}{2} =) 1\frac{3}{4} \text{ or } \frac{7}{4}$$

oe

M1

$$\text{their } 1\frac{3}{4} \div 2 \div 2 \text{ or their } \frac{7}{4} \div 2$$

oe

M1dep

$$\frac{7}{8}$$

oe fraction

A1

Alternative method 4

$$(1.25 - 0.5 =) 0.75$$

or

$$(1.25 + 0.5 =) 1.75$$

accept equivalent in percentages but must see % sign

M1

$$(0.5 + 0.75 \div 2 =) 0.875$$

or

$$(1.25 - 0.75 \div 2 =) 0.875$$

or

$$\left(\frac{1.25 + 0.5}{2} =\right) 0.875$$

or 87.5%

0.875 must be correct

accept equivalent in percentages but must see % sign

M1dep

$$\frac{7}{8}$$

oe fraction

A1

Alternative method 5

Positions of $\frac{1}{2}$ and $1\frac{1}{4}$ correctly marked on line

or

correct midpoint marked on line

if more points are marked, labels of $\frac{1}{2}$ and $1\frac{1}{4}$ must be given or indicated

mark intention in terms of exact position accept decimals or equivalent fractions

Correct midpoint marked on line

and

$\frac{3}{4}$ marked as $\frac{6}{8}$ and 1 marked as $\frac{8}{8}$

oe fractions with common denominator > 4

M1dep

$$\frac{7}{8}$$

oe fraction

A1

Additional Guidance

In alternative method 5: $\frac{1}{4}$ marked at $1\frac{1}{4}$ is sufficient for $1\frac{1}{4}$

In all schemes, award of M1dep means that M2 is awarded

Use the scheme that gives the greatest number of marks – ignore errors in the scheme(s) you do not use

[3]

Q17.

A correct pair of fractions meeting all conditions

eg $\frac{1}{9}$ and $\frac{2}{9}$ or $\frac{1}{12}$ and $\frac{1}{4}$

B2

a pair of fractions which add to $\frac{1}{3}$ but which do not satisfy all conditions

eg, $\frac{1}{6}$ and $\frac{1}{6}$ or $\frac{2}{3}$ and $-\frac{1}{3}$

or $\frac{1}{3}$ – any fraction less than $\frac{1}{3}$ correctly

changed to common denominator with at least one numerator correct

B1

$\frac{1}{3}$ changed to any equivalent fraction

$\frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \dots$

or

$\frac{1}{3}$ – any fraction less than $\frac{1}{3}$

B3

[3]

Q18.

$\frac{5}{18}$

oe eg $\frac{10}{36}$

B1

Additional Guidance

Mark the answer line. If this is blank, mark the working

Allow 0.277... (minimum two 7s and two dots) or correct notation for recurring decimals

If values are given in one or more forms, either on the answer line or in working with nothing on the answer line, all values must be correct

eg1 $\frac{5}{18} = 0.277\dots$ on answer line

B1

eg2 $\frac{5}{18}$ and 0.518 in working with answer line blank

B0

$1\frac{2}{3}$ or $\frac{1.66\dots}{6}$

without answer in correct form

B0

[1]

Q19.

$$\frac{11}{4} \text{ or } \frac{16}{9}$$

oe fraction

M1

$$\frac{\text{their } 11 \times \text{their } 16}{4 \times 9} \text{ or } \frac{176}{36}$$

oe fraction

$$\frac{11 \times 8}{2 \times 9} \text{ or } \frac{88}{18} \text{ or } \frac{11 \times 4}{9} \text{ or } \frac{44}{9}$$

M1dep

$$4\frac{8}{9}$$

oe mixed number

SC2 4.8

A1

Additional Guidance

$$4\frac{16}{18} \text{ or } 4\frac{32}{36}$$

M1M1A1

Working in decimals is SC2 or 0

[3]

Q20.

Valid common denominator for subtraction with at least one numerator correct

M1

$$\text{eg } \frac{21}{30} - \frac{8}{30} \text{ or } \frac{13}{30}$$

$$\frac{105}{150} - \frac{40}{150} \text{ or } \frac{65}{150}$$

condone decimals in numerator(s)

$$\text{their } \frac{13}{30} \times \frac{3}{2}$$

$$\text{their } 13 \div 2$$

$$\text{or their } 30 \div 3$$

oe product

$$\frac{13}{30}$$

their $\frac{13}{30}$ can be any single fraction, mixed number or decimal other than

$$\frac{13}{30} \text{ inverted or } \frac{7}{10} \text{ or } \frac{4}{15}$$

condone decimals in numerator(s)

correct answer not in correct fraction

$$\frac{6.5}{10}$$

form eg $\frac{6.5}{10}$ scores M1M1

M1

$$\frac{13}{20} \text{ or } \frac{39}{60}$$

oe fraction

$$\frac{29}{20}$$

SC2 $\frac{29}{20}$ oe fraction or mixed number

A1

Additional Guidance

If 10 or 15 is used as the common denominator, both numerators must be correct for the first mark

Correct fraction in working with incorrectly simplified fraction on answer line

M2A1

Correct fraction in working with conversion to decimal on answer line

M2A0

$$\frac{65}{150} \div \frac{2}{3} = \frac{32}{50}$$

M1M0A0

$$\frac{65}{150} \div \frac{2}{3} = \frac{32.5}{50} \text{ with no further working}$$

M1M1A0

[3]

Q21.

$$\frac{33}{8}$$

or equivalent fraction

B1

$$4\frac{1}{8}$$

ft their improper fraction correctly converted to a mixed number

answer only of $4\frac{1}{8}$ scores B1B1

B1ft

Additional Guidance

If their initial answer is a proper fraction they cannot access the second mark

eg $\frac{3}{8} \times 11 = \frac{33}{88}$

B0B0ft

If their ft mixed number can be simplified, the simplification is not required for the second mark

eg $\frac{3}{8} \times 11 = \frac{44}{8} = 5\frac{4}{8}$

B0B1ft

$0.375 \times 11 = 4.125$

B1B0

$33 \div 8$

B0B0

$33 \div 8 = 4\frac{1}{8}$

B1B1

$\frac{11}{8} = 1\frac{3}{8}$ then $1\frac{3}{8} \times 3 = 3\frac{9}{8}$ (this gets first B1) $= 4\frac{1}{8}$

B1B1

[2]

Q22.

- (a) Division set up, with 8 and a remainder 3 seen in correct position
or
 $830 \leq \text{answer} < 840$ but not 834

$$\begin{array}{r} 8 \\ 8 \overline{)91374} \end{array} \quad \text{or} \quad \begin{array}{r} 8 \\ 9 \ 1 \ 7 \ 4 \\ \underline{8 \ 8} \\ 3 \end{array}$$

M1

834

A1

Additional Guidance

Build up method or chunking method must lead to $830 \leq \text{answer} < 840$ to score M1 or better

(b) $\frac{35}{42} (+) \frac{18}{42}$

oe

fractions with a correct common denominator and at least one correct numerator

M1

$$\frac{53}{42}$$

oe improper fraction

A1

$$1 \frac{11}{42}$$

oe mixed number

ft for correct conversion of an improper fraction to a mixed number

B1ft

Additional Guidance

For B1ft the mixed number must not be an integer

Beware $5 + 3 = 53$

M0

When attempts are made to cancel the fraction, full marks cannot be scored

$$\frac{53}{42} = \frac{9}{4} = 2 \frac{1}{4} \quad (\text{attempt to cancel occurs before conversion to mixed number})$$

M1A1B0

$$\frac{53}{42} = 1 \frac{11}{42} = 1 \frac{1}{3} \quad (\text{attempt to cancel occurs after completely correct answer seen})$$

M1A1B0

[5]

Q23.

210 – 90 or 120

M1

their $120 \div 4$

oe

M1dep

30(.00)

A1

Q24.

Alternative method 1

$$-2\frac{7}{8} + 15\frac{1}{4} \quad \text{oe}$$

$$\text{or } 15\frac{2}{8}$$

common denominator for both fractional parts of the mixed numbers

$$\text{or } (-)2.875 \text{ and } 15.25$$

conversion of both numbers to decimals with at least one correct

$$\text{or } (-)\frac{23}{8} \text{ and } \frac{61}{4}$$

conversion of both numbers to improper fractions with at least one correct

M1

$$-2\frac{7}{8} + 15\frac{2}{8}$$

oe common denominator

$$\text{or } -2.875 + 15.25$$

correct decimals

$$\text{or } \frac{23}{8} + \frac{122}{8}$$

oe common denominator

M1dep

$$\frac{99}{8} \text{ or } 12\frac{3}{8} \text{ or } 12.375$$

oe fraction, mixed number or decimal

A1**Alternative method 2**

$$-2 + 15 \text{ and } (-)\frac{7}{8} + \frac{1}{4}$$

M1

$$-2 + 15 \text{ and } (-)\frac{7}{8} + \frac{2}{8}$$

oe common denominator

$$\text{or } 13 - \frac{5}{8}$$

M1dep

$$\frac{99}{8} \text{ or } 12\frac{3}{8} \text{ or } 12.375$$

oe fraction, mixed number or decimal

A1**Additional Guidance**

$$15\frac{1}{4} - -2\frac{7}{8} \text{ scores M0, but followed by } 15\frac{2}{8} + 2\frac{7}{8} \text{ scores M1 on Alt 1}$$

Values in 2nd mark must be correct; no ft from incorrect conversion

$$\frac{99}{8} \text{ incorrectly converted to a decimal or mixed number}$$

M1M1A1

$$13\frac{-5}{8}$$

M1M1A0

[3]

Q25.

(a) $\frac{2}{5}$

B1

(b) $\frac{5}{9} \times 72$ or 8×5 or $360 \div 9$

oe e.g. multiples of 8 listed and 5th one chosen with maximum one error

M1

40

SC1 32

A1

Additional Guidance

$$\frac{40}{72}$$

M1A0

40 out of 72

M1A1

[3]

Q26.

2160

may be implied by 240 or 10 800

B1

5 × their 2160

9

or 5×240 or $10\,800 \div 9$

or 1200

oe

M1

1473

A1

Additional Guidance

Accept 0.55 or 0.56 or better for $\frac{5}{9}$

eg $\frac{5}{9}$ (2160) + 273 (no indication that they know to multiply by $\frac{5}{9}$)

B1M0A0

eg $\frac{5}{9} \times (2160) + 273$

B1M1A0

eg 2130, $5 \times 2130 \div 9$

B0M1A0

[3]

Q27.

1

5 in top centre cell

1 in centre cell

1

10 in bottom right cell

oe decimals

B2 any two correct

or

the product of the centre column and the diagonal from top left to bottom right are both 1

B1 any one correct

or

the product of the centre column or the diagonal from top left to bottom right is 1

B3

Additional Guidance

A response can be awarded B2 if it meets both ways of scoring B1

Eg one correct value **and** the product of the centre column is 1 (see example below right)

10	$\frac{1}{15}$	$\frac{1}{2}$
$\frac{1}{20}$	3	20
2	5	$\frac{1}{30}$

Diagonal
and centre
column
each have
product 1

10	$\frac{1}{50}$	$\frac{1}{2}$
$\frac{1}{20}$	10	20
2	5	$\frac{1}{10}$

Bottom
right cell
correct
and centre
column
= 1

or

B2

Centre column has product 1

10	$\frac{1}{50}$	$\frac{1}{2}$
$\frac{1}{20}$	10	20
2	5	$\frac{2}{10}$

Diagonal has product 1

10	$\frac{1}{20}$	$\frac{1}{2}$
$\frac{1}{20}$	8	20
2	5	$\frac{1}{80}$

or

B1

[3]

Q28.

Two fractions less than 1 with product $\frac{3}{10}$
 $\frac{3}{5}$ and $\frac{1}{2}$ or $\frac{6}{10}$ and $\frac{5}{10}$
either order

B1

Additional Guidance

Accept negatives if each processed fraction is less than 1

eg $-\frac{3}{2}$ and $-\frac{1}{5}$

B1

eg $-\frac{1}{2}$ and $\frac{3}{5}$

B1

eg $-\frac{1}{2}$ and $\frac{3}{5}$

B0

Do not accept decimals within the fractions eg $\frac{0.6}{1}$ and $\frac{0.5}{1}$

B0

$\frac{11}{10}$ and $\frac{3}{11}$

B0

$\frac{3}{10}$ and $\frac{1}{1}$

B0

0.6 and 0.5

B0

[1]

Q29.

$\frac{28}{9}$

B1

[1]